

Exercise 26

What is the geometric relation between the vectors \mathbf{v} and \mathbf{w} if $\|\mathbf{v} \times \mathbf{w}\| = \frac{1}{2}\|\mathbf{v}\|\|\mathbf{w}\|$?

Solution

The magnitude of the cross product of \mathbf{v} and \mathbf{w} is

$$\|\mathbf{v} \times \mathbf{w}\| = \|\mathbf{v}\|\|\mathbf{w}\| \sin \theta,$$

where θ is the angle between \mathbf{v} and \mathbf{w} . Comparing this and the provided equation,

$$\sin \theta = \frac{1}{2},$$

which means the angle between the vectors is

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} = 30^\circ.$$